

Model of the mixed state of type-II superconductors in high magnetic fields

I. L. Landau^{†‡} and H. R. Ott[†]

[†] Laboratorium für Festkörperphysik, ETH Hönggerberg, CH-8093 Zürich, Switzerland

[‡] Kapitza Institute for Physical Problems, 117334 Moscow, Russia

Abstract. In superconductors with large values of the Ginzburg-Landau parameter κ , exposed to magnetic fields close to the upper critical field H_{c2} , the magnetic field is practically homogeneous across the sample and the density of supercurrents is negligibly small. In this case, there is no obvious reason for the formation of Abrikosov vortices, characteristic for the commonly known mixed state. We consider an alternative model for describing the mixed state for $\kappa \gg 1$ and magnetic fields close to H_{c2} . We argue that with decreasing magnetic field the traditional vortex structure is adopted via a first order phase transition, revealed by discontinuities in the magnetization as well as the resistivity.

PACS numbers: 74.60.Ec

It is commonly accepted that the mixed state of a type-II superconductor is characterized by the penetration of an external magnetic field into the sample along quantized vortex lines or vortices. The spatial extent of the superconducting order parameter $\psi(r)$ near a vortex axis is determined by circular currents flowing around the vortex line. The velocity of the relevant charge carriers, i.e., the Cooper pairs, diverges at the center of the vortex line (see, e.g., Ref. [1]), thus reducing the superconducting order parameter progressively until it vanishes on the vortex axis. In most cases the vortices may thus be considered as thin normal filaments embedded in a superconducting environment. A completely different situation may, however, be established in superconductors with a Ginzburg-Landau (G-L) parameter $\kappa \gg 1$. If the applied magnetic field is close to the upper critical field H_{c2} , the distance between adjacent vortex cores is much smaller than the magnetic field penetration depth $\lambda(T)$. In this case, there is practically no expulsion of the magnetic field from superconducting regions and the density of shielding currents is negligibly small. These circumstances are naturally unfavorable for the formation of common Abrikosov vortices and hence the character of the mixed state in magnetic fields close to H_{c2} may be very different from that adopted at lower fields.

In this paper we consider that in magnetic fields close to H_{c2} , the natural alternative to the conventional vortex structure is the formation of superconducting filaments, embedded in the matrix of the normal metal. The properties of such superconducting filaments may be analyzed by numerically solving the G-L equations [2]. First, we consider a single cylindrical superconducting filament in an infinitely extended normal metal. The magnetic field inside the normal metal is oriented parallel to the filament and its value is set to $H_0 < H_{c2}$. For the numerical analysis we have chosen cylindrical coordinates (r, ϕ, z) with the z -axis parallel to the filament and $r = 0$ at its center. The G-L equations for an infinitely long cylindrical filament

$$-\frac{1}{\kappa^2} \left(\frac{d^2\psi}{dr^2} + \frac{1}{r} \frac{d\psi}{dr} \right) + (A^2 - 1) \psi + \psi^3 = 0 \quad (1)$$

and

$$\frac{d}{dr} \left(\frac{dA}{dr} + \frac{A}{r} \right) = A\psi^2. \quad (2)$$

may thus be solved in one-dimension, with all quantities depending only on the radial coordinate r . Here, ψ is the superconducting order parameter normalized by its equilibrium value in a bulk superconductor; A is the vector potential of the magnetic field expressed in units of $\sqrt{2}H_c\lambda(T)$, where H_c is the thermodynamic critical field. The coordinate r is measured in units of $\lambda(T)$.

The boundary conditions are set at the center of the filament ($r = 0$). For the superconducting order parameter $\psi(r)$, we have $d\psi/dr|_{r=0} = 0$ and $\psi(0) = \text{const.}$ For the vector potential $A(r)$, we have $A(0) = 0$ and $\text{curl } A|_{r=0} = H(0)$, the value of the magnetic field at the center of the filament. We have checked that the solutions with $A(0) = 0$, equivalent to a vanishing supercurrent at $r = 0$, correspond to a minimum of the free energy. By integration of the G-L equations with these boundary conditions, we obtain $\psi(r)$, $A(r)$ and the distribution of the magnetic field $H(r)$ around the filament. We also calculated the Gibbs free energy F_{fil} of the filament per unit length. For a single filament, F_{fil} reaches its minimum if, at the boundary with the normal metal, the order parameter vanishes with a zero derivative, i.e., $\psi(r) \rightarrow 0$ and $d\psi/dr \rightarrow 0$ for $r \rightarrow \infty$.

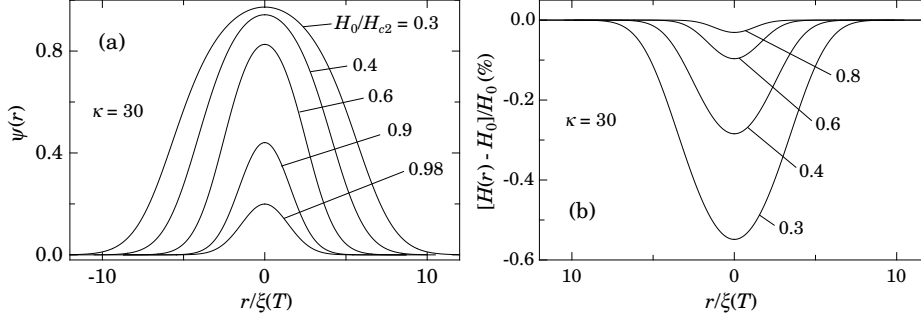


Figure 1. Radial variation of (a) the normalized order parameter $\psi(r)$ and (b) the magnetic field for different values of H_0/H_{c2} . $\xi(T) = \lambda(T)/\kappa$ is the G-L coherence length.

The calculations have been made for 3 values of $\kappa = 10, 30$, and 100 . Fig. 1(a) shows the calculated profiles of the normalized superconducting order parameter $\psi(r)$ for one filament and, in Fig. 1(b), corresponding profiles of the magnetic field have been plotted. For all κ values between 10 and 100 and $H_0/H_{c2} \geq 0.3$, these profiles are independent of κ . In this approach, all characteristics of the filament are uniquely determined by the value of the applied magnetic field H_0 .

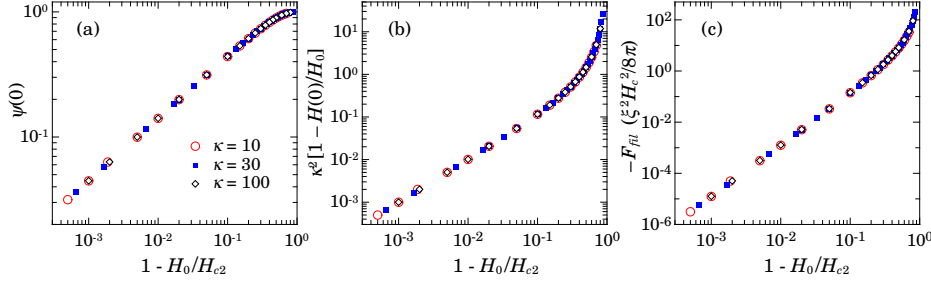


Figure 2. Different parameters of a single superconducting filament as functions of $(1 - H_0/H_{c2})$. (a) The amplitude of the normalized superconducting order parameter at the center of the filament. (b) The normalized difference between the magnetic field at the center of the filament and H_0 , multiplied by κ^2 . (c) The Gibbs free energy of the filament F_{fil} (per unit of length) in units of $\xi^2(T)H_c^2/8\pi$ with respect to that of the normal metal in a magnetic field equal to H_0 .

In Figs. 2(a-c) we display various quantities, plotted versus $(1 - H_0/H_{c2})$. Figure 2(a) shows the amplitude of the normalized G-L order parameter at the center of the filament; in Fig. 2(b) we have plotted the product $\kappa^2[1 - H(0)/H_0]$, and in Fig. 2(c) we show the free energy of the filament F_{fil} . It may be seen that while $\psi(0)$ and F_{fil} are independent of κ , the expulsion of the magnetic field from the center of the filament, $[1 - H(0)/H_0]$, is inversely proportional to κ^2 , i.e., the magnetic moment of the filament rapidly decreases with increasing κ . According to Fig. 2(c) the free energy of the filament with respect to that of the normal metal is negative and it vanishes at $H_0 = H_{c2}$. This obvious but gratifying result demonstrates that the superconducting phase can only be stable in magnetic fields below H_{c2} .

So far, we have considered a single superconducting filament in a normal metal matrix and we now consider the interaction between the filaments. Because the current density in the normal metal between the filaments is zero there is no long-range interaction between them and the contribution to the free energy due to filament-filament interaction is zero unless the neighboring filaments are very close to each other. In order to show that the short-range interaction between the filaments is repulsive, we introduce the velocity of superconducting electrons $v_s = j_s/(2e\psi^2)$, where j_s is the supercurrent, c is the speed of light, and e is the electron charge. In our case v_s increases with the distance from the axis of the filament approximately proportionally to r . The velocities v_s arising from the neighboring filaments have opposite directions in the space between them. Because v_s cannot have discontinuities in the superconducting phase, the filaments cannot merge, but must always be divided by a boundary where the order parameter $\psi \equiv 0$ [4]. The situation in the boundary region is very similar to that arising in the center of the Abrikosov vortex where v_s also changes its sign, demanding that $\psi = 0$ along the vortex axis.

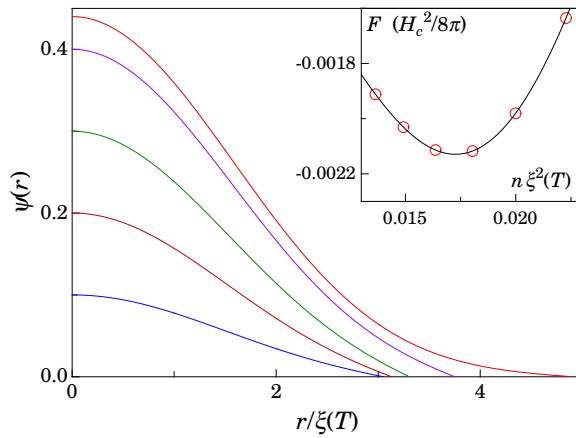


Figure 3. Profiles of the order parameter for different values of $\psi(0)$ and $H_0/H_{c2} = 0.9$. In this case, as well as for a single filament, these profiles are independent of κ for $10 < \kappa < 100$. The inset shows the total free energy of the system of filaments $F = n\tilde{F}_{fil}$ as a function of their density. The solid line is a guide to the eye.

Because the number of filaments in the sample is only limited by a short-range repulsion, they are expected to form a rather dense triangular configuration as is illustrated in the inset to Fig 4(b). In order to analyze the resulting profile of an individual filament, we again use Eqs. (1) and (2) but, in order to satisfy the condition $\psi = 0$ between the filaments, the boundary condition $d\psi/dr|_{r \rightarrow \infty} = 0$ is abandoned. Several solutions of the G-L equations are shown in Fig. 3. In this case, the order parameter vanishes with a non-zero derivative and we may introduce the radius of the filament as the value of $r = D/2$ at the point where $\psi = 0$.

As may be seen in Fig. 3, the diameter D of the filament decreases with decreasing value of $\psi(0)$. The important consequence of the G-L equations is that the diameter D of the filament cannot be smaller than a certain minimal value D_{\min} . The only solution of the G-L equations for $D \leq D_{\min}$ is $\psi(r) \equiv 0$. The dependence of D_{\min} on

$(1 - H_0/H_{c2})$ is shown in Fig. 4a. We note that also for superconducting lamellae a minimal thickness δ_{\min} exists. The dependence of δ_{\min} on the applied magnetic field is shown in Fig. 4a, as well. Both D_{\min} and δ_{\min} decrease with decreasing magnetic field. It has been shown that $\delta_{\min}|_{H_0=0} = \pi\xi(T)$ [5]. The corresponding value of $D_{\min}|_{H_0=0} \approx 4.81\xi(T)$.

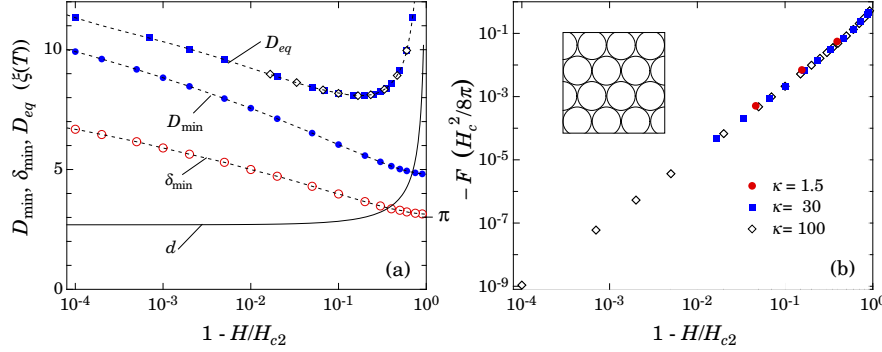


Figure 4. (a) The minimal thickness of superconducting lamellae, δ_{\min} , the minimal diameter of the superconducting filament, D_{\min} , and the equilibrium value D_{eq} versus $(1 - H_0/H_{c2})$. The period d of the vortex structure is shown as the solid line. The dashed lines are guides to the eye. The Gibbs free energy F of the system of filaments (per unit of volume) with respect to that of the normal metal in a magnetic field equal to H_0 . The inset represents the triangular lattice of superconducting filaments

In the following we assume that the distance between the filaments is equal to their diameter D as is shown in the inset to Fig. 4(b). For a triangular lattice, the density n of filaments is thus $n = 2/(\sqrt{3}D^2)$. In order to evaluate the equilibrium density of filaments, we plot the total free energy of the system of filaments $F = n\tilde{F}_{fil}$, where \tilde{F} is the Gibbs free energy calculated for solutions of the type shown in Fig. 3, versus n as is illustrated in the inset to Fig. 3. The position of the minimum on this curve corresponds to the equilibrium value of n . The magnetic field dependence of the equilibrium diameter D_{eq} of the filaments is presented in Fig. 4(a). As may be seen, D_{eq} , which is the period of the triangular lattice of the filaments, is considerably larger than the equivalent quantity d for the vortex structure.

The dependencies of $\psi(0)$ and $(1 - H(0)/H_0)$ on the magnetic field for an equilibrium filament configuration practically coincide with those calculated in the single-filament approximation for $H_0/H_{c2} \geq 0.5$ (see Figs. 2(a) and 2(b)).

Fig. 4(b) shows the free energy of the system of filaments as a function of $(1 - H_0/H_{c2})$. The energy was calculated assuming that the filaments are cylindrical. We have to adopt this approach, in order to use one-dimensional G-L equations which are the basis of our consideration. It is obvious, however, that, because of their mutual interaction, the filaments should adopt a hexagonal rather than a circular cross-section. We note, however, that this simplification may only result in free energy values that are slightly higher than the actual ones.

As we have argued, the system of quantized vortices is not the only way to realize the mixed state of type-II superconductors which may also be established by an ensemble of superconducting filaments. At low magnetic fields, where the equilibrium

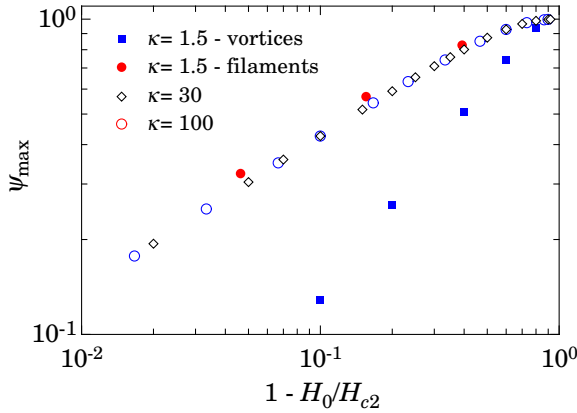


Figure 5. The maximum amplitude of the order parameter for the equilibrium configuration of filaments and for the vortex lattice. The data for vortices is taken from Ref. [6].

diameter of filaments $D_{eq} \geq \lambda(T)$, the filaments are unstable against the formation of a vortex line in the center of the filament. In order to identify the stable arrangement in higher fields, the free energies for these two configurations should be compared. To our knowledge the only study in which the two-dimensional G-L equations have been solved for the vortex lattice is presented in Ref. [6]. Fig. 5 shows the results of our calculations of the order parameter amplitude ψ_{\max} versus $(1 - H_0/H_{c2})$ for filaments, together with the results of Ref. [6]. In the high-magnetic-field limit, the term proportional to the magnetic moment in the Gibbs free energy may be neglected and the density of the free energy reduces to $F \approx -\psi^4(r)H_c^2/8\pi$. As may clearly be seen in Fig. 5, the amplitude of ψ for the vortex lattice is considerably smaller than that for the filaments and the ratio $\psi_{\max}(\text{filaments})/\psi_{\max}(\text{vortices})$ increases with increasing magnetic field. Thus, in sufficiently high magnetic fields, the free energy for the configuration of superconducting filaments is expected to be lower than that for the vortex lattice.

The properties of a mixed state consisting of superconducting filaments are quite different from those of the Abrikosov vortices. First, because the filaments are always separated by normal conducting regions, the sample resistance for currents perpendicular to the direction of the magnetic field never vanishes and the true zero-resistance superconducting state may be achieved only after the transition to the vortex structure. Second, in the case of filaments, the magnetic flux faces no barriers to move in or out of the sample and the magnetization of the sample must be reversible, independent of whether the filaments are pinned or not.

The analysis presented in this paper shows that, in magnetic fields close to H_{c2} , the mixed state may well consist of a triangular lattice of superconducting filaments separated by regions where the superconducting order parameter $\psi = 0$. With decreasing external magnetic field, the configuration of superconducting filaments necessarily has to undergo a transition to the conventional mixed state, involving Abrikosov vortices. The value of the transition field is determined by the free-energy balance between these two configurations which cannot be determined without more precise calculations of the free energy for both cases. The transition from one type

of mixed state to the other involves a complete change of topology and must be accompanied by discontinuities in both the resistivity and the magnetic moment of the sample. We also expect some hysteresis, as well as a latent heat, dictated by the discontinuity of the magnetization. In other words, this transition is expected to exhibit all the features of a first-order phase transition. Such transitions are observed in high temperature superconductors at $H < H_{c2}$ and are usually attributed to the melting of the vortex lattice [7, 8, 9, 10].

We wish to thank R. Monnier for numerous stimulating discussions.

References

- [1] A. A. Abrikosov, *Fundamentals of the theory of metals*. North-Holland, Amsterdam, 1988.
- [2] The calculation procedure is described in Ref. 3. The only difference is that here we use the Ginzburg-Landau equations written in cylindrical coordinates.
- [3] A. Cossy-Favre, B. Dutoit, E. Holguin, I. Landau, and L. Rinderer, J. of Low Temp. Phys. **94**, 619 (1994).
- [4] The fact that superconducting domains in the mixed state of type-II superconductor cannot merge into one, may even more clearly be demonstrated if we consider two superconducting planes instead of filaments. We choose the vector-potential $A = 0$ along the boundary between the planes and the order parameter in its complex form. In this case v_s is proportional to $\nabla\phi$, where ϕ is the phase of the order parameter. Because $\nabla\phi$ changes its sign if we go from one superconducting plane to the other, it is clear that the order parameters of these two planes cannot be forced to merge. Superconducting planes, however, are unstable against the formation of a system of Abrikosov vortices between them. The distance L between the vortices may be found from the condition $L\nabla\phi = \pi$. In the case of superconducting filaments, such boundary vortices cannot be formed because the phase difference $\Delta\phi$ along the filament-filament boundary is much less than π .
- [5] A. F. Andreev and P. Tekel, Zh. Eksp. Teor. Fiz. **76**, 1740 (1972) [Sov. Phys. JETP **35**, 807 (1972)].
- [6] E. H. Brandt, Phys. Rev. Lett. **78**, 2208 (1997).
- [7] H. Pastoriza, M. F. Goffman, A. Arribère, and F. de la Cruz, Phys. Rev. Lett. **72**, 2951 (1994).
- [8] R. Liang, D. A. Bonn, and W. N. Hardy, Phys. Rev. Lett. **76**, 835 (1996).
- [9] U. Welp, J. A. Fendrich, W. K. Kwok, G. W. Crabtree, and B. W. Veal, Phys. Rev. Lett. **76**, 4809 (1996).
- [10] A. Schilling, R. A. Fisher, N. E. Phillips, U. Welp, D. Dasgupta, W. K. Kwok, and G. W. Crabtree, Nature (London) **382**, 701 (1996).
- [11] R. Roulin, A. Junod, and E. Walker, Science **273**, 1210 (1996).